# Experimental DREAM MCMC: A 1D Bayesian Back Analysis Method Extension

## Abstract

This document summarises the development and implementation of a Bayesian back-analysis framework for one-dimensional consolidation measurements in geotechnical engineering, based Kelly & Huang (2015) proof of concept. The work involved creating a Python-based Markov chain Monte Carlo (MCMC) simulation using DREAM program package (Vrugt, 2016) to update two key geotechnical parameters being the coefficient of volume compressibility and coefficient of vertical consolidation with progressive settlement observations. The code was iteratively refined to handle numerical stability, visualise results, and conduct experiments on prediction improvement analogous to a construction program. All changes were limited to the progress\_SETT.py file for the experiment, demonstrating convergence to true values after approximately 7 observations.

## Introduction

The observational method (Peck, 1969) is used in geotechnical engineering for monitoring performance during construction of embankments on soft soils. Kelly & Huang (2015) demonstrated that Bayesian updating can enhance this method by systematically incorporating measurements to refine model parameters and subsequent predictions for settlement and excess pore pressure. This provide a robust statistical framework for dealing with uncertainty, allowing timely decision to be made on both risk and opportunity.

The goal of this work was to implement a Python based script that replicates and extents the Kelly & Huang (2015) case study using the DiffeRential Evolution Adaptive Metropolis (DREAM) MCMC algorithm (Vrugt, 2016). The benefits of this algorithm and its use specifically for updating key geotechnical soft soil parameters is outlined by Jones *et al.* (2025).

The Python script was developed to:

* Use the governing consolidation equation from Eq (4) of Kelly & Huang (2015) based on Atkinson (2007).
* Incorporate normal distributions for key parameters
* Performa progressive Markov chain Monte Carlo simulations with increasing observations
* Visualise updates and analyses the prediction accuracy and convergence

The development process involved implementation of the governing consolidation equation, improvement of numerical stability and implementation of appropriate diagnosis and plots for traceability.

## Methodolgy

### Geotechnical model and governing equation

The numerical model is based on Terzaghi’s one dimensional consolidation theory for a single elastic later with one-way drainage with the top drained or permeable and base undrained or impermeable. The geometry used includes:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Model Parameters used to generate artificial measurement set | | | | |
| Variable | Description | Value | Units | Comment |
| H | Layer thickness | 5.5 | m | Soft soil |
| Hf | Fill thickness | 3 | m | constant |
|  | Unit weight of fill | 22 | kN/m3 | constant |
|  | coef. of volume compressibility | 0.0014 | 1/kPa | Prior informed |
|  | coef. of vertical consolidation | 80 | m2/year | Prior informed |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| Variable | Units | Adopted Distribution | Mean, | COV | STD, |
|  | 1/kPa | Lognormal | 0.001 | 0.4 | 0.0004 |
| H | m | Lognormal | 5 | 0.1 | 0.5 |
|  | kN/m3 | Lognormal | 20 | 0.1 | 2 |
|  | m2/year | Lognormal | 40 | 1 | 40 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | | |  | (1) |
| where: | | , |  |  | | |

There parameters follow normal distributions with coefficients of variation as described in Kelly & Huang (2015).

A diagram of a physics equation

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### MCMC implementation

* DREAM toolbox: Used Vrugt’s DRAM for Bayesian inference with Gaussian likelihood.
* Parameters: 4 parameters (, *H,* , ) with latin hypercube initialisation and reflect bounds
* Experiment: Progressive updates with 1 to 10 observations, running separate MCMC for each subset
* Diagnosis: Added plots for scale reduction (Gelman *et al.*, 2004), acceptance rate, marginal densities, trace plots and autocorrection.

### Experiment

An experiment was conducted to assess prediction improvement of a single layers consolidation model based on progressive observation:

* Prior prediction using mean parameters
* Progressive MCMC with cumulative observation (1 to 10)
* Prediction curves generation for full range after each update
* Error calculated at t - 1.3 years relative to the ‘true’ settlement (0.508m)
* Threshold for ‘reasonable closeness’ set at < 5 % error

### Updated Paramters

The results showed convergence at 7 observations (error 0.4%) with updated posterior parameters of = 0.0016 1/kPa and = 76.6 m2/year.

## Code Development

The code progress\_SETT.py was developed as a stand alone Python script that implements the following:

* Forward Model: Implementation of governing consolidation equation with numerical clipping for stability
* Priors: Normal distribution for all parameters
* MCMC loop: Progressive runs with global scope for observation count
* Visualizations:
  + Figure 1: Progressive predictions up to n observations
  + Figure 2: overlay of all predictions with convergence highlights
  + Diagnosis: GR diagnosis, acceptance rate and traceplots
* Output: Console, textfil and .PNG results summary
* Error Handling: Synatx errors fixed, overflows and broadcasting issues through scoping and slicing (OpenAI, 2025).

All changes were self contained within progress\_SETT.py. The concept extends the work of Kelly & Huang (2015). Debugging, refinement and implementation was assisted by xAI (2025) and OpenAI (2025).

## Results

🎉 PROGRESSIVE SETTLEMENT PREDICTION RESULTS - KELLY & HUANG 2015

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With 1 observations:

Posterior m\_v: 0.0013, c\_v: 59.7

Predicted settlement at t=1.3 years: 0.355 m

True settlement at t=1.3 years: 0.508 m

**Error: 30.2%**

With 2 observations:

Posterior m\_v: 0.0014, c\_v: 67.5

Predicted settlement at t=1.3 years: 0.408 m

True settlement at t=1.3 years: 0.508 m

**Error: 19.7%**

With 3 observations:

Posterior m\_v: 0.0015, c\_v: 72.2

Predicted settlement at t=1.3 years: 0.417 m

True settlement at t=1.3 years: 0.508 m

**Error: 17.9%**

With 4 observations:

Posterior m\_v: 0.0016, c\_v: 74.9

Predicted settlement at t=1.3 years: 0.449 m

True settlement at t=1.3 years: 0.508 m

**Error: 11.7%**

With 5 observations:

Posterior m\_v: 0.0016, c\_v: 74.7

Predicted settlement at t=1.3 years: 0.485 m

True settlement at t=1.3 years: 0.508 m

**Error: 4.5%**

✅ Prediction is reasonably close (<5%) with 5 observations!

With 6 observations:

Posterior m\_v: 0.0016, c\_v: 74.2

Predicted settlement at t=1.3 years: 0.505 m

True settlement at t=1.3 years: 0.508 m

**Error: 0.6%**

With 7 observations:

Posterior m\_v: 0.0016, c\_v: 76.5

Predicted settlement at t=1.3 years: 0.516 m

True settlement at t=1.3 years: 0.508 m

**Error: 1.5%**

With 8 observations:

Posterior m\_v: 0.0016, c\_v: 77.4

Predicted settlement at t=1.3 years: 0.520 m

True settlement at t=1.3 years: 0.508 m

**Error: 2.4%**

With 9 observations:

Posterior m\_v: 0.0016, c\_v: 76.5

Predicted settlement at t=1.3 years: 0.525 m

True settlement at t=1.3 years: 0.508 m

**Error: 3.4%**

With 10 observations:

Posterior m\_v: 0.0017, c\_v: 78.1

Predicted settlement at t=1.3 years: 0.528 m

True settlement at t=1.3 years: 0.508 m

**Error: 3.8%**

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Results: The prediction improved with more data, conversion to true values

## Conclusion

This implementation demonstrates the useful power of Bayesian inference using the DREAM MCMC algorithm for updating key geotechnical parameters. It provides a practical tool for a mathematical implementation of the observational method within a rigorous and traceable framework enabling accurate and timely predictions to be made. This synthetic analysis only converged after a significant amount of consolidation had occurred. Additional parameters and use of real data would improve the proposed method. Future extensions would need to demonstrate convergence at less than 50 % consolidation as discussed by Kelly & Huang (2015) for this to become a useful in practice.

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Figure 1. Progressive Settlement Predictions

A group of graphs and charts

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Figure 2. Bayesian updating schematic showing the settlement prediction after less than 5% error convergence achieved (5 observations) with Posterior statistics and the 95 % credible interval shown. Posterior for Cv and Mv shown.

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Figure 3. Summary of progressive settlement predictions showing the decrease in error with increase in the number of observations.

# Code Review Summary for progress\_SETT.py

## OVERVIEW

The updated progress\_SETT.py script implements a **Bayesian back-analysis framework** for one-dimensional consolidation settlement data following the methodology of *Kelly & Huang (2015)*. It progressively updates key soil parameters using the **DREAM (DiffeRential Evolution Adaptive Metropolis)** MCMC algorithm, performing sequential analyses with increasing numbers of settlement observations.

The new version introduces **automated figure generation and saving**, enhanced **posterior convergence tracking**, and structured visualization through three figures (Figures 1–3). It also preserves an **adapter-based interface** for compatibility with the DREAM\_Suite module, eliminating issues previously caused by global variable scoping.

## **KEY COMPONENTS**

### 2.2.1 Imports and Setup

* Imports scientific libraries (numpy, matplotlib, scipy.stats) for computation and plotting, and os, sys for file handling.
* Automatically sets the working directory to the script’s location and adds the DREAM\_Suite path to the system import list.
* Ensures all figures and outputs are saved directly to the working directory (e.g. E:\Python\2015\_KELLY\_MCMC).

### Forward Model

* **kelly\_core** – Implements the consolidation equation (Eq. 4) from Kelly & Huang (2015) for computing settlement *s(t)* based on parameters [ mᵥ, H, γ\_f, cᵥ ].  
  It applies a truncated series for numerical stability and allows flexible truncation via n\_obs and t\_obs\_full.
* **kellyFORTRAN** – A single-argument adapter that enables DREAM\_Suite to call the model. It delegates internally to kelly\_core using temporary adapter variables \_ADAPTER\_N\_OBS and \_T\_OBS\_FULL, ensuring a clean interface without global scope conflicts.

### Data and Settings

* Defines observed settlements (D\_obs) and time steps (t\_obs\_full), with N\_FULL = 10.
* Specifies reference (“true”) parameters true\_theta = [0.0014, 5.5, 22.0, 80.0] to generate a benchmark curve.
* Configures DREAM algorithm controls (DREAMPar) and prior statistics from *Kelly & Huang* Table 2.  
  Parameter bounds are set to ±3 standard deviations around the prior mean.

### Experiment Logic

* **run\_mcmc(n\_obs)**  
  Runs the DREAM MCMC for each incremental observation set.  
  Initializes the adapter variables, executes the simulation, extracts posterior samples (after burn-in), computes the posterior mean, and predicts the full settlement curve.  
  A thinned sample array is also returned for credible-interval and histogram plotting.
* Iterates sequentially from 1 to 10 observations, storing posterior means, predictions, and thinned samples for later visualization.

#### Figure 1 – Progressive Settlement Predictions

* Displays the true settlement, prior prediction, and **progressive DREAM-updated predictions** for each observation count (1 → 10).
* Uses a **viridis** colormap for smooth progression, a **logarithmic x-axis**, and an **inverted y-axis** (convention for settlement).
* Includes a compact two-column legend for clarity.
* **Automatically saved** as Figure\_1.png in the working directory.

#### Figure 2 – Four-Panel Bayesian Summary

Figure 2 visualizes the **first MCMC run achieving < 5 % error convergence** (in this case, 5 observations). Posterior statistics and the 95 % credible interval (CI) are computed from the posterior samples of that run.

| **Panel** | **Description** |
| --- | --- |
| **Top Left – Bayesian Update** | Observed data vs. prior and posterior settlement curves, including a 95 % CI envelope. Shows the model performance at convergence. |
| **Top Right – Embankment Schematic** | Modern 2025-style gradient cross-section illustrating the drained and impermeable boundaries, stress arrow (σ), and the prior → posterior parameter updates. |
| **Bottom Left – Posterior of cᵥ** | Histogram of the posterior distribution of cᵥ (m²/yr) with prior, posterior, and true values marked. |
| **Bottom Right – Posterior of mᵥ** | Histogram of the posterior distribution of mᵥ (1/kPa) with prior, posterior, and true markers. |

* The script automatically identifies the convergence run (error < 5 %) and labels the figure title with n\_obs = 5.
* **Saved automatically** as Figure\_2.png.

#### Figure 3 – Textual Results Summary

* Opens and reads progress\_SETT\_results.txt, rendering the text contents as a formatted **monospaced figure** for archival.
* Displays and saves the image as Figure\_3\_progress\_SETT\_results.png.
* Provides automatic error handling if the text file cannot be opened.

## ANALYSIS AND OUTPUT

* Computes the settlement prediction error at *t = 1.3 years* for each MCMC stage.
* Prints results to the console and writes them to progress\_SETT\_results.txt.
* Defines **convergence** as the first case where absolute error < 5 %.
* Tracks this index for use in Figure 2 and clearly annotates convergence within the text output.

## UNDERSTANDING AND OBSERVATIONS

### Strengths

* The **adapter pattern** eliminates global-variable conflicts, ensuring clean integration with DREAM\_Suite.
* Three automated, high-quality figures provide visual, statistical, and textual transparency for publication or peer review.
* Dynamic identification of the 5 % convergence run automates posterior selection for Figure 2.
* Thinned posterior sampling enhances computational efficiency and stability.

### Assumptions

* Assumes that DREAM\_Suite correctly calls kellyFORTRAN(theta) and implements a Gaussian likelihood.
* Priors (means and COVs) are taken from *Kelly & Huang (2015)* Table 2 and may require modification for other case studies.

### Limitations

* Each observation count executes a full, independent MCMC run, which can be computationally expensive.
* The schematic is static and does not dynamically update with progressive stages.
* No built-in exception handling for failed DREAM convergence or incomplete chains.

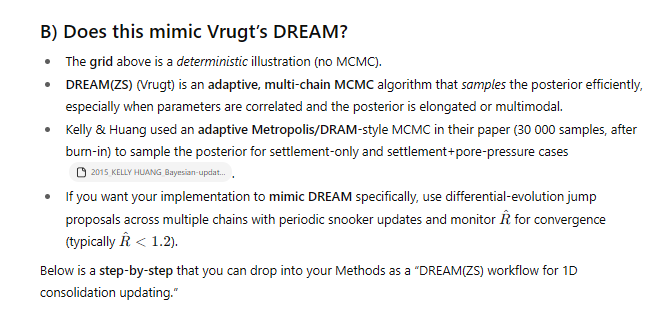
### Potential Improvements

* Incorporating **sequential MCMC** updating rather than restarting for each n\_obs.
* Adding dynamic visual updates or posterior trace diagnostics for convergence assessment.
* Extending the schematic to include excess pore-pressure evolution.

This version of progress\_SETT.py delivers a robust, fully automated Bayesian consolidation analysis framework that replicates *Kelly & Huang (2015)* while enhancing interpretability through structured visual outputs and reproducible documentation.

*(Prepared 18 October 2025, 07:50 PM AEDT).*

*Previous example attempt - however good DREAM summary*



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A screenshot of a computer

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A screenshot of a math problem

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